

# Quantum quench within the gapless phase of the spin-1/2 Heisenberg XXZ spin-chain

Mario Collura<sup>1</sup>, Pasquale Calabrese<sup>1</sup> and Fabian H. L. Essler<sup>1,2</sup>

<sup>1</sup> SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy.

<sup>2</sup> The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford, OX1 3NP, United Kingdom.

**Abstract.** We consider an interaction quench in the critical spin-1/2 Heisenberg XXZ chain. We numerically compute the time evolution of the two-point correlation functions of spin operators in the thermodynamic limit and compare the results to predictions obtained in the framework of the Luttinger liquid approximation. We find that the transverse correlation function  $\langle S_j^x S_{j+\ell}^x \rangle$  agrees with the Luttinger model prediction to a surprising level of accuracy. The agreement for the longitudinal two-point function  $\langle S_j^z S_{j+\ell}^z \rangle$  is found to be much poorer. We speculate that this difference between transverse and longitudinal correlations has its origin in the locality properties of the respective spin operator with respect to the underlying fermionic modes.

## 1. Introduction

The non-equilibrium dynamics of isolated quantum systems represents a theoretical and experimental challenge that raises numerous fundamental questions with applications to a variety of fields of modern physics. In a so-called *interaction quench*, a system evolves unitarily from an initial state, which is the ground-state of a translationally invariant Hamiltonian differing from the one governing the time evolution by an experimentally tunable interaction parameter [1, 2]. Over the last few years it has become clear that the quench dynamics in integrable systems is very unusual. While generic systems relax locally to standard Gibbs distributions at effective temperatures set by the initial energy density [3, 4, 5], integrable models retain a detailed memory of the initial state even at infinite times [6, 7, 8, 9] by virtue of having infinite sets of conserved charges. It has been conjectured that in integrable systems expectation values of local operators in the stationary state can be calculated using a generalized Gibbs ensemble (GGE)[7], a statistical ensemble formed from all local conserved charges (the role of locality of the integrals of motion has been highlighted in Refs [10, 11]). In order to understand and describe the behaviour of the quench dynamics in interacting integrable models new approaches have been developed [9, 12, 13, 14, 15, 16]. Investigations of quench dynamics in such models have focussed on the one-dimensional Bose gas (i.e. the Lieb-Liniger model) [17, 18, 19, 20, 21], the XXZ spin-chain [22, 23, 24, 25, 26], the Richardson model [27] and sine- and sinh-Gordon field theories [28, 16]. For the Bose gas, the knowledge of the overlaps between the BEC state and the eigenstates of the system with arbitrary interaction [19, 29] allowed for a characterization of the stationary state [19] which turned out to be compatible with a properly constructed GGE [30], while the full time evolution of observables is analytically known only in the limit of strong interaction [31, 32] (but it is possible to exploit integrability to study the time evolution numerically [9, 19, 33]). For the XXZ spin chain, the overlaps with the Bethe eigenstates are known only for some classes of product states [34, 35, 36, 37, 38]. Again these overlaps allowed for a description of the stationary state in the gapped regime [24, 25] which, surprisingly, turned out to disagree with the predictions of a GGE formed by the infinite number of ultra-local conservation laws of the XXZ chain [22, 23]. This disagreement motivated many investigations aimed at understanding its origin, see e.g. [39, 40, 41, 42, 43, 44], which culminated in the recently proposed solution [45] based on taking into account quasi-local conserved charges.

In spite of this intense research effort, relatively little is known about the full time dependence of observables in interacting integrable models in the thermodynamic limit. This is in stark contrast to non-interacting theories (or models that map onto such) [15, 31, 32, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61]. In particular, there are no exact results for the behaviour after quantum quenches in the gapless phase of the Heisenberg XXZ model (with the exception of the XX chain, corresponding to non-interacting fermions [62, 63]). However, such quenches have been analyzed in a number of numerical works [62, 64, 65, 66]. The critical phase of the Heisenberg chain is

of particular interest, because in equilibrium many of its properties can be described by Luttinger liquid theory [67, 68, 69, 70]. An obvious question is then, to what extent, if any, Luttinger liquid theory can be used for the analysis of quench dynamics. Given that the quantum quench deposits an extensive amount of energy in the system, perturbations to the Luttinger liquid description as well as cutoff effects are expected to play a role. Before considering such issues one first ought to investigate whether the simple results obtained for quantum quench in the Luttinger model [46] can effectively describe the non-equilibrium dynamics of the spin chain, at least in some parameter regimes and/or specific time windows. Indeed this question has been addressed in Refs [71, 65] for the so-called Z-factor and in Ref. [66] for the stationary values of correlation functions in momentum space.

Effects of perturbations away from the Luttinger model have been analyzed by renormalization group methods [72, 73]. This gives some insight in how perturbations lead to eventual thermalization on very long time scales. We mention that many other aspects of the quench dynamics of the Luttinger model have been considered in the literature [74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89]. The aim of our work is to quantify in some detail to what extent “simple” Luttinger liquid theory can be used to describe the time evolution of local observables after an interaction quench in the critical phase of the spin-1/2 Heisenberg XXZ chain. To that end we compare predictions of Luttinger liquid theory for spin-spin correlation functions to numerical results.

### 1.1. The model and the quench protocol

We consider is the one-dimensional spin-1/2 XXZ chain with Hamiltonian

$$H(\Delta) = J \sum_{j=1}^L (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z), \quad (1)$$

where  $S_j^\alpha$  are spin operators at site  $j$  of a one dimensional chain and  $\Delta$  parametrizes the exchange anisotropy. Throughout this paper we will set  $J = 1$ , which amounts to measuring all energies in units of  $J$ . The equilibrium properties of the system in the thermodynamic limit are well known: for  $|\Delta| < 1$  the system is quantum critical, while for  $|\Delta| > 1$  the ground state is antiferromagnetically ordered and excitations acquire an energy gap.

In the following we want to study the out-of-equilibrium dynamics in the gapless phase. Our protocol is as follows: we prepare the system in the ground state  $|\Psi_0\rangle$  of the XX model ( $\Delta_0 = 0$ ) and at time  $t = 0$  suddenly quench the anisotropy parameter  $\Delta$  to a finite value in the interval  $(-1, 1]$  and then time evolve unitarily with Hamiltonian  $H(\Delta)$ . This protocol is often referred to as *interaction quench*. We note that the generalization of our work to an arbitrary quench  $\Delta_0 \rightarrow \Delta$  of the anisotropy parameter within the gapless phase is straightforward. In the following we present a systematic numerical study of the quench dynamics and compare the numerical results to analytic expressions for correlation functions obtained via Luttinger liquid theory (LL).

### 1.2. Organization of the manuscript

In Sec. 2 we review the bosonization of the XXZ Hamiltonian and summarize how to use Luttinger liquid theory to obtain results for long-distance behaviour of spin-spin correlation functions after an interaction quench. In Sec. 3 we present the results of numerical computations of the same quantities by means the time-evolving block decimation algorithm. The numerical results are carefully compared with the bosonization predictions. Finally, in Sec. 6 we summarize our results.

## 2. Bosonization of the XXZ spin chain

In this section we summarize some key results on the low energy field theory description of the spin-1/2 XXZ chain in zero magnetic field, details can be found in e.g. [68]. At low energies the spin-1/2 Heisenberg chain can be described as a free compact boson perturbed by irrelevant operators

$$\mathcal{H}(\Delta) = \frac{v}{2} \int dx \left[ K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 \right] + \mathcal{H}_{\text{irr}}. \quad (2)$$

Here  $\phi = \varphi + \bar{\varphi}$  is a canonical Bose field,  $\theta = \varphi - \bar{\varphi}$  the dual field ( $\varphi$  and  $\bar{\varphi}$  are chiral components),  $v$  is the spin velocity,  $K$  is the so-called Luttinger parameter, and  $\mathcal{H}_{\text{irr}}$  denotes an infinite tower of perturbing operators that are irrelevant in the renormalization group sense. In zero field the values of  $K$  and  $v = \tilde{v}a_0$  (we recall that we have set  $J = 1$ ) are known exactly from the Bethe ansatz solution of the XXZ chain [90]

$$\tilde{v} = \frac{\pi \sqrt{1 - \Delta^2}}{2 \arccos \Delta}, \quad K = \frac{\pi}{2} \frac{1}{\pi - \arccos \Delta}, \quad (3)$$

where  $a_0$  is the lattice spacing. In the XX limit  $\Delta = 0$  we have  $\tilde{v} = 1$  and  $K = 1$ . The bosonized expressions for the lattice spin operators are [68, 69, 70]

$$S_j^z \simeq m - \frac{a_0}{\sqrt{\pi}} \partial_x \phi(x) + (-1)^j a_1 \sin(\sqrt{4\pi} \phi(x)) + \dots, \quad (4)$$

$$S_j^x \simeq b_0 (-1)^j \cos(\sqrt{\pi} \theta(x)) + i b_1 \sin(\sqrt{\pi} \theta(x)) \sin(\sqrt{4\pi} \phi(x)) + \dots, \quad (5)$$

where  $x = ja_0$  and  $m = \langle S_j^z \rangle$ . Using these expressions one arrives to the equilibrium two-point functions

$$\langle S_j^x S_{j+\ell}^x \rangle = (-1)^\ell \frac{A_0^x}{\ell^{1/(2K)}} - A_1^x \frac{1}{\ell^{2K+1/(2K)}}, \quad (6)$$

$$\langle S_j^z S_{j+\ell}^z \rangle = m^2 - \frac{2K}{\pi^2} \frac{1}{\ell^2} + A_1^z \frac{(-1)^\ell}{\ell^{2K}}. \quad (7)$$

In the absence of a magnetic field ( $m = 0$ ) the various non-universal constants are known. In two-point functions the relevant amplitudes are

$$A_0^x = \left| \frac{b_0^2}{2} \right|, \quad A_1^x = \left| \frac{b_1^2}{4} \right|, \quad A_1^z = \left| \frac{a_1^2}{2} \right|. \quad (8)$$

Their values are given by [69]

$$A_0^x = \frac{1}{8(1-\eta)^2} \left[ \frac{\Gamma(\frac{\eta}{2(1-\eta)})}{2\sqrt{\pi}\Gamma(\frac{1}{2(1-\eta)})} \right]^\eta \exp \left[ - \int_0^\infty \frac{dt}{t} \left( \frac{\sinh(\eta t)}{\sinh(t) \cosh[(1-\eta)t]} - \eta e^{-2t} \right) \right], \quad (9)$$

$$A_1^x = \frac{1}{2\eta(1-\eta)} \left[ \frac{\Gamma(\frac{\eta}{2(1-\eta)})}{2\sqrt{\pi}\Gamma(\frac{1}{2(1-\eta)})} \right]^{\eta+\frac{1}{\eta}} \times \exp \left[ - \int_0^\infty \frac{dt}{t} \left( \frac{\cosh(2\eta t)e^{-2t} - 1}{2\sinh(\eta t) \sinh(t) \cosh[(1-\eta)t]} + \frac{1}{\sinh(\eta t)} - \frac{\eta^2 + 1}{\eta} e^{-2t} \right) \right], \quad (10)$$

$$A_1^z = \frac{2}{\pi^2} \left[ \frac{\Gamma(\frac{\eta}{2(1-\eta)})}{2\sqrt{\pi}\Gamma(\frac{1}{2(1-\eta)})} \right]^{\frac{1}{\eta}} \exp \left[ \int_0^\infty \frac{dt}{t} \left( \frac{\sinh[(2\eta-1)t]}{\sinh(\eta t) \cosh[(1-\eta)t]} - \frac{2\eta-1}{\eta} e^{-2t} \right) \right]. \quad (11)$$

Here the parameter  $\eta$  is related to the anisotropy  $\Delta$  by

$$\eta = 1 - \frac{1}{\pi} \arccos(\Delta) = \frac{1}{2K}. \quad (12)$$

### 2.1. Interaction quench

Our quench protocol is to prepare our spin chain in the ground state of the XX-chain, i.e. the Hamiltonian (1) with  $\Delta = 0$ , and then time evolve the system with  $H(\Delta)$  at times  $t > 0$ . This corresponds to a sudden quench of the interaction strength from  $\Delta = 0$  to  $\Delta \in (-1, 1]$  at time  $t = 0$ . Assuming that a projection to the low-energy description in terms of a Luttinger liquid is possible, this corresponds to preparing our system in the ground state of the free boson theory  $\mathcal{H}(\Delta = 0)$ , and then time evolve it with  $\mathcal{H}(\Delta)$ . The observables of interest are the low-energy projections (4), (5) of the spin operators. Two point functions of these operators have been calculated for interaction quenches in the Luttinger model in Refs. [46, 47], and can be used in the case of interest here. In order to make our discussion self-contained we summarize the main points of the necessary calculations in Appendix A. The final results of these calculations are

$$\langle S_j^x(t) S_{j+\ell}^x(t) \rangle \simeq (-1)^\ell \frac{A^x}{\sqrt{\ell}} \left| \frac{1}{(2\tilde{v}t)^2} \frac{\ell^2 - (2\tilde{v}t)^2}{\ell^2} \right|^{(1/K^2-1)/8} + \dots, \quad (13)$$

and

$$\begin{aligned} \langle S_j^z(t) S_{j+\ell}^z(t) \rangle &\simeq B_z \left\{ -\frac{1-K^2}{8\pi^2} \left[ \frac{1}{(\ell+2\tilde{v}t)^2} + \frac{1}{(\ell-2\tilde{v}t)^2} \right] - \frac{1+K^2}{4\pi^2} \frac{1}{\ell^2} \right\} \\ &+ A^z \frac{(-1)^\ell}{\ell^2} \left| \frac{1}{(2\tilde{v}t)^2} \frac{\ell^2 - (2\tilde{v}t)^2}{\ell^2} \right|^{(K^2-1)/2} + \dots. \end{aligned} \quad (14)$$

In the correlator  $\langle S_j^x(t) S_{j+\ell}^x(t) \rangle$  we retain only the staggered term because the other contributions turn out to be small, while for  $\langle S_j^z(t) S_{j+\ell}^z(t) \rangle$  the smooth and staggered terms turn out to be comparable in magnitude. As first pointed out in Ref. [46] for the Luttinger model, the power-law decays (13), (14) are very different from their equilibrium analogs. The amplitudes  $A^{x/z}$  and  $B^z$  are non-universal and will be fixed

$\Delta$	$K$	$v$	$\alpha$	$\beta$
-0.5	3/2	$3\sqrt{3}/8$	-5/36	5/4
-0.2	1.14704	0.868468	-0.0599861	0.315694
0	1	1	0	0
0.2	0.886377	1.12386	0.0682023	-0.214336
0.5	3/4	$3\sqrt{3}/4$	7/36	-7/16

**Table 1.** Luttinger liquid parameters for the values of  $\Delta$  considered here.

below by fitting (13), (14) to numerical results. They are related to the equilibrium amplitudes  $A_0^x$  and  $A_z^1$  via unknown relations involving the cutoffs of pre- and post-quench Hamiltonians. If the quench does not require a cutoff adjustment one would expect that

$$A^x \sim A_0^x, \quad A^z \sim A_1^z, \quad B^z \sim 1. \quad (15)$$

We will see that these relations turn out to be approximately fulfilled, with deviations of the order of a few percent.

As (13) and (14) are derived in the framework of a field theory approximation they are applicable to the description of lattice correlators only as long as  $\ell, vt, |\ell - 2vt| \gg a_0 = 1$ . In Table 1 we summarize the numerical values of the parameters for the post-quench interaction strengths that we will consider in the following, i.e.  $\Delta = -0.5, -0.2, 0.2, 0.5$ .

### 3. Quantum quenches in the spin-1/2 XXZ chain: from free fermions to the quantum critical phase

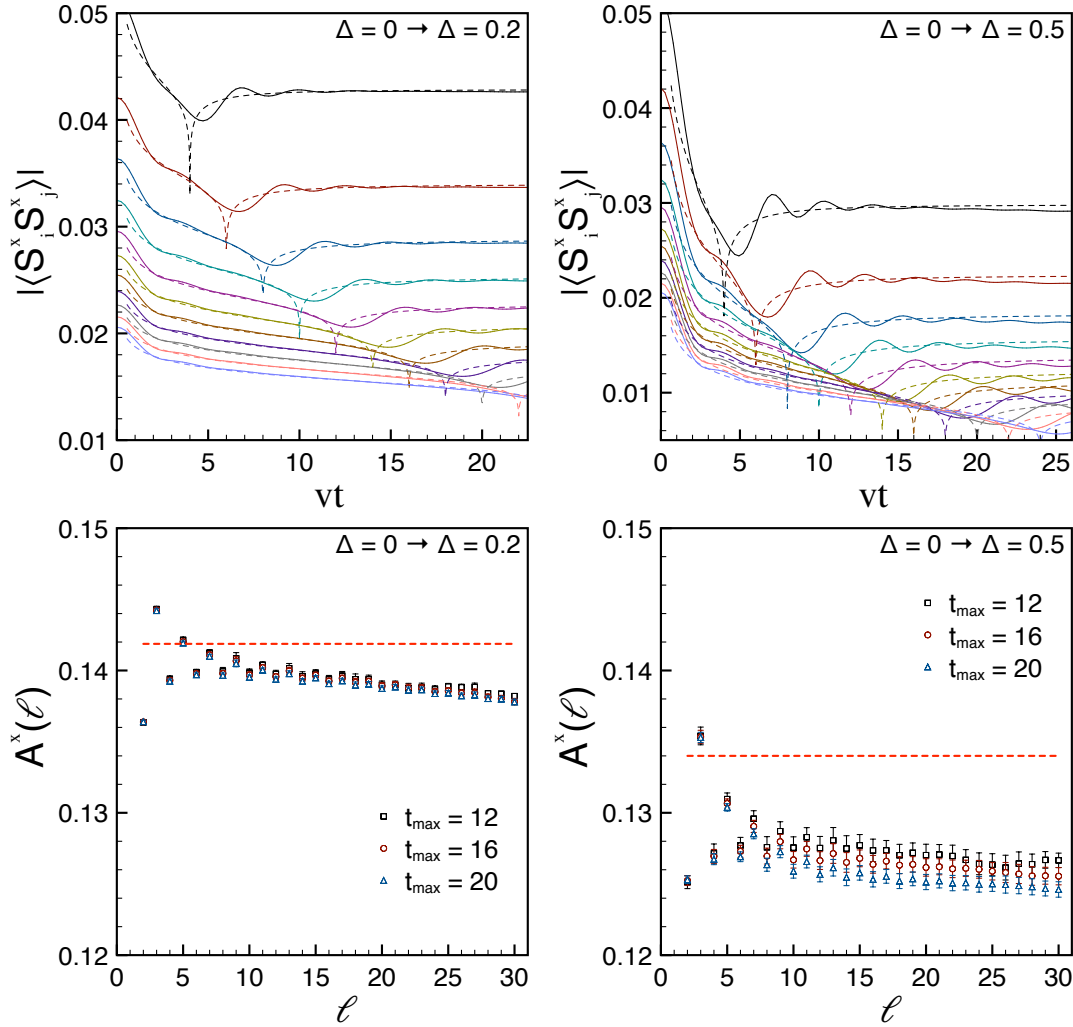
We now turn to the central part of this paper, a detailed numerical study of the quench dynamics of the XXZ spin-chain from  $\Delta_0 = 0$  to a final  $-1 < \Delta \leq 1$  in the gapless phase. To this end we employ the infinite time-evolving block decimation (iTEBD) algorithm [91] to study the dynamics induced by the post quench Hamiltonian (1). The algorithm is based on a matrix product state (MPS) description of one-dimensional lattice models and works directly in the thermodynamic limit. Compared to other algorithms like time-dependent density matrix renormalization group [92], the iTEBD has the great advantage of not introducing systematic errors due to finite size effects, since it works directly in the thermodynamic limit. This is made possible by two main features: (i) invariance under translation of the Hamiltonian; (ii) the possibility to parallelize the local updates of the time-evolving block decimation procedure.

#### 3.1. Method

The iTEBD algorithm we are using is composed of two different parts: the first obtains an accurate MPS description of the initial state (in our case the ground state of the XX Hamiltonian) and the second deals with the time evolution.

The ground state of the XX Hamiltonian  $H_{XX} \equiv H(\Delta = 0)$  is found using the iTEBD algorithm in imaginary time. The infinite chain is prepared in a given, simple state  $|\Psi_s\rangle$ , which we choose as  $|\Psi_s\rangle = \bigotimes_{j \in \mathbb{Z}} (|\uparrow\rangle_j + |\downarrow\rangle_j)/\sqrt{2}$ . This state has a trivial MPS representation of bond dimension  $\chi_0 = 1$ . We then evolve  $|\Psi_s\rangle$  in imaginary time with  $\exp(-\tau H_{XX})$ , implemented using the second order Suzuki-Trotter decomposition with imaginary time-step  $\tau = 0.01$ . We verified that further decreasing  $\tau$  does not affect the final result within our numerical accuracy. As is well known, since the imaginary-time evolution operator is not unitary, the MPS loses its canonical form (i.e. its normalisation is not constant). Hence this form must be restored while time elapses and in particular before taking the expectation value of any operator. We control the convergence of the imaginary time algorithm by keeping track of the energy density  $E_0$  and waiting for it to become stationary (with an accuracy of  $10^{-16}$ ). We repeat this procedure for several values of the bond dimension up to  $\chi_0 = 128$ . A check of this procedure is provided by our best estimate of the ground-state energy density, which is  $E_0 = -0.31830981$ . This differs from the exact value  $E_{XX} = -1/\pi$  by  $7 \cdot 10^{-8}$ . This is quite satisfactory as the XX model is gapless and exhibits long range correlations. An exact MPS representation of the ground state in this case requires an infinite bond dimension (in other words, the singular values involved in the MPS show an algebraic decay). For our purposes the description of the initial state with  $\chi_0 = 128$  is sufficiently accurate. Indeed, comparing the longitudinal and transverse spin-spin correlation functions with the exact analytical results up to distances of  $\ell = 50$ , we find an extremely good agreement, with relative errors smaller than 3% and 1% for respectively longitudinal and transverse correlators.

Using this MPS as our initial state, we can address the real time evolution. We again use a second order Suzuki-Trotter decomposition of the evolution operator  $\exp(-idtH)$  with time step  $dt = 0.05$  (we verified that the data are not affected by the time discretisation). We adapt the number of states used to describe the reduced Hilbert space by retaining, at each time step, all Schmidt vectors corresponding to singular values larger than  $\lambda_{min} = 10^{-12}$ , up to a maximum value  $\chi_{MAX} = 1024$ . The latter is actually reached fairly quickly, reflecting the fast growth of the entanglement entropy under time evolution. Indeed, it is well known that the computational complexity of the time evolution of a quantum system using algorithms based on MPS descriptions is essentially set by the growth of the bipartite entanglement. As the entanglement increases with time, we have to enlarge the dimension  $\chi$  of the reduced Hilbert space in order to optimally control the truncation error. In spite of our refined adaptive choice of  $\chi$ , the truncation procedure is still the main source of error in our algorithm. For this reason, we are able to reach a maximum time  $T = 20$  without significant truncation error (the Schmidt error coming from the iterative truncation of the Hilbert space remains always smaller than  $2 \cdot 10^{-3}$ ). This is also reflected on the behaviour of the entanglement entropy of half system, which grows linearly for all explored times as it should after a global quantum quench [93, 54].



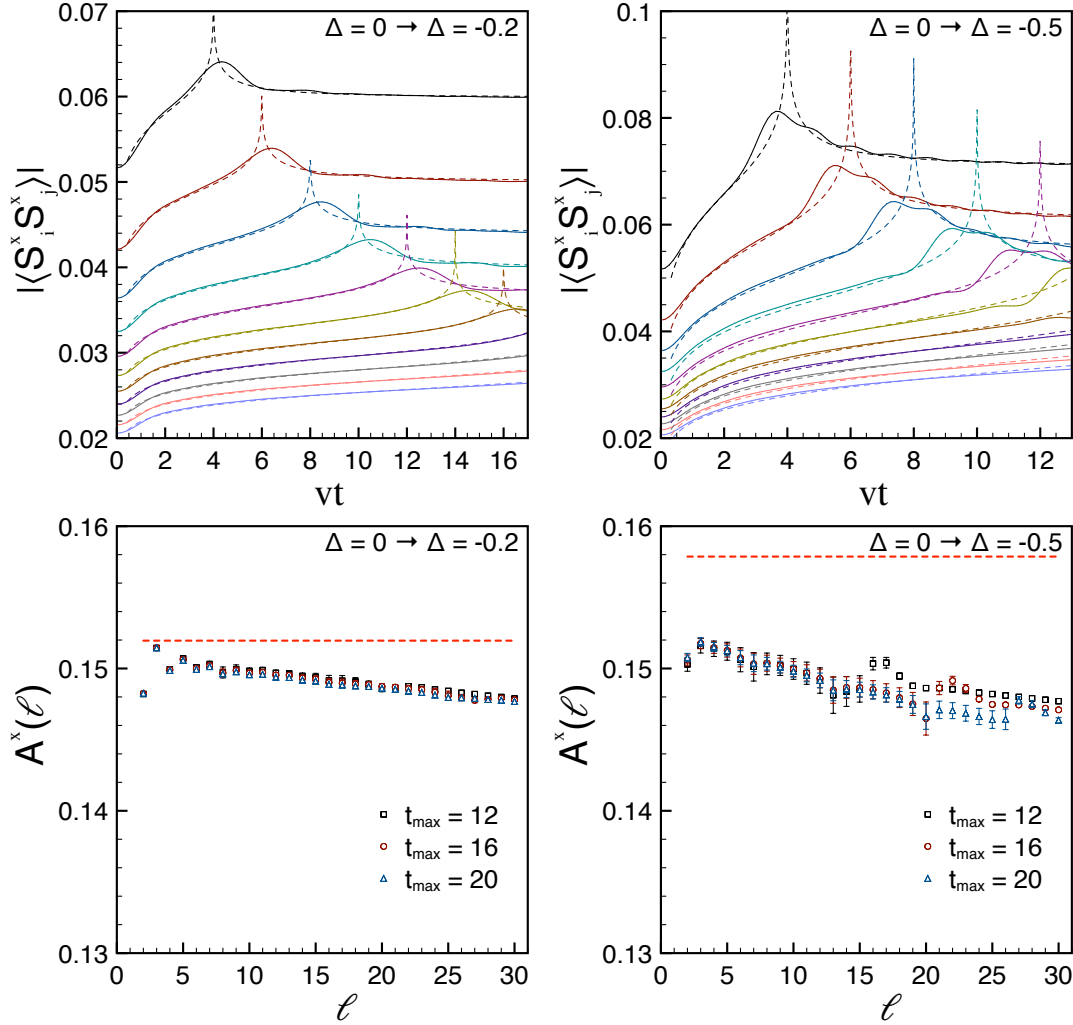
**Figure 1.** (Top)  $|\langle S_i^x S_j^x \rangle|$  vs time, for different distances  $\ell = j - i = 8, 12, 16, \dots, 48$  (from top to bottom) and interaction strengths  $\Delta > 0$ . Full lines are the numerical data, dashed lines represent the best fits to (13) on the interval  $0 \leq t < t_{\max} = 20$ . (Bottom) The pre-factor  $A^x$  obtained from the best fit for different values of  $t_{\max}$ . The dashed line is the equilibrium value of the amplitude  $A_0^x$  in Eq. (9).

#### 4. Transverse correlation functions $\langle S_i^x S_j^x \rangle$

We now turn to the two point correlation function of the x-component of spin. We find that  $\langle S_j^x S_{j+\ell}^x \rangle$  displays a strongly alternating behaviour in space, i.e. it is dominated by a contribution of the form  $(-1)^\ell f(\ell)$ , where  $f(\ell)$  is a smooth function. Given that this structure is correctly predicted by the Luttinger model, cf. (13), we will only analyze the absolute value  $|\langle S_j^x S_{j+\ell}^x \rangle|$  (as this makes plotting the correlator easier).

Our numerical results for  $|\langle S_j^x S_{j+\ell}^x \rangle|$  are shown in Figs 1 and 2 as functions of time for different, fixed distances  $\ell$ , and in Fig. 3 as functions of  $\ell$  for the largest time  $t = 20$ . The main qualitative features of this correlator are as follows. (i) Apart from a brief, non universal transient,  $|\langle S_j^x S_{j+\ell}^x \rangle|$  is surprisingly well described by the Luttinger liquid

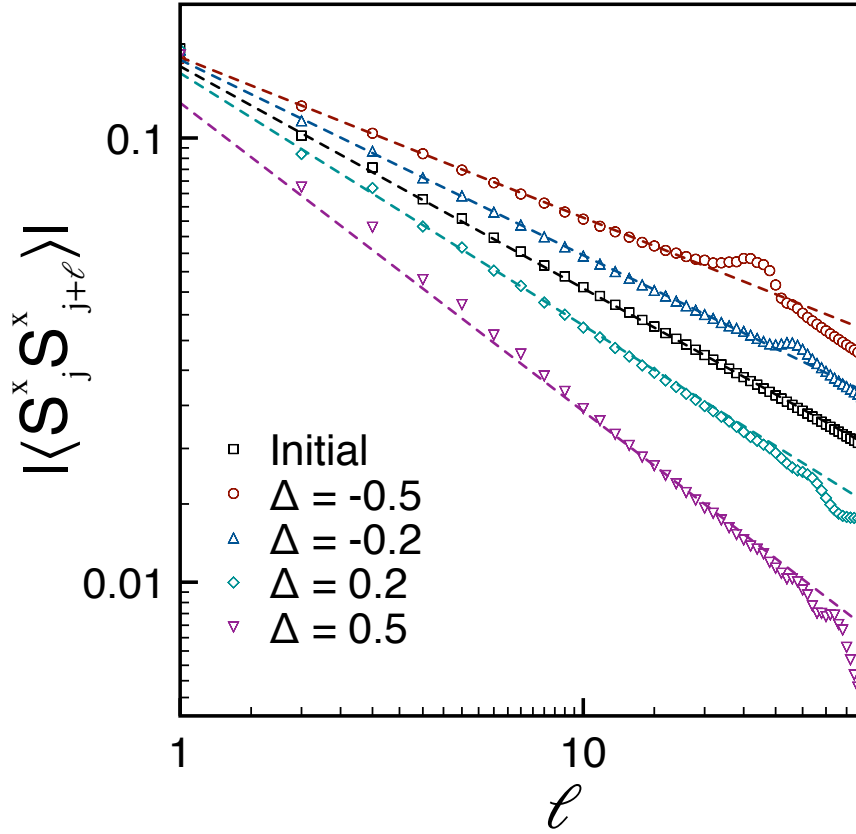




**Figure 2.** The same as in Figure 1 for  $\Delta < 0$ .

prediction (13).

(ii) For all quenches the correlator exhibits a clear light-cone effect. At “short” times  $1 \ll 2vt \ll \ell$ , the correlations decay in a power law fashion  $\sim t^{-\alpha}$  with exponent  $\alpha = (1/K^2 - 1)/4$ , in agreement with (13). We note that the exponent  $\alpha$  is related to properties of the  $Z$ -factor studied in Ref. [65]. The exponent  $\alpha$  is positive (negative) for positive (negative) interactions  $\Delta$  and indeed a qualitatively different behaviour is evident just by looking at Figs. 1 and 2. For late times  $2vt > \ell$  the correlation function exhibits a power-law decay in time towards its asymptotic stationary value, which scales with the distance as  $\ell^{-\alpha-1/2}$  (cf. Eq. (13)). The algebraic decay as a function of distance at late times is shown in Fig. 3 for several quenches. Another feature visible in Fig. 3 is the interaction strength dependence of the velocity, at which correlations spread. This is reflected in the presence of “bumps” in the spatial dependence of correlation functions in Fig. 3 at the positions of the light cones  $\ell^* \simeq 2vt$ . Inspection of the numerical results for different values of  $\Delta$  shows that  $\ell^*$ , and hence  $v$ , depends on  $\Delta$ . At distances  $\ell > \ell^*$ , the correlators still display the power-law decay  $\ell^{-1/2}$  of the initial state. In particular



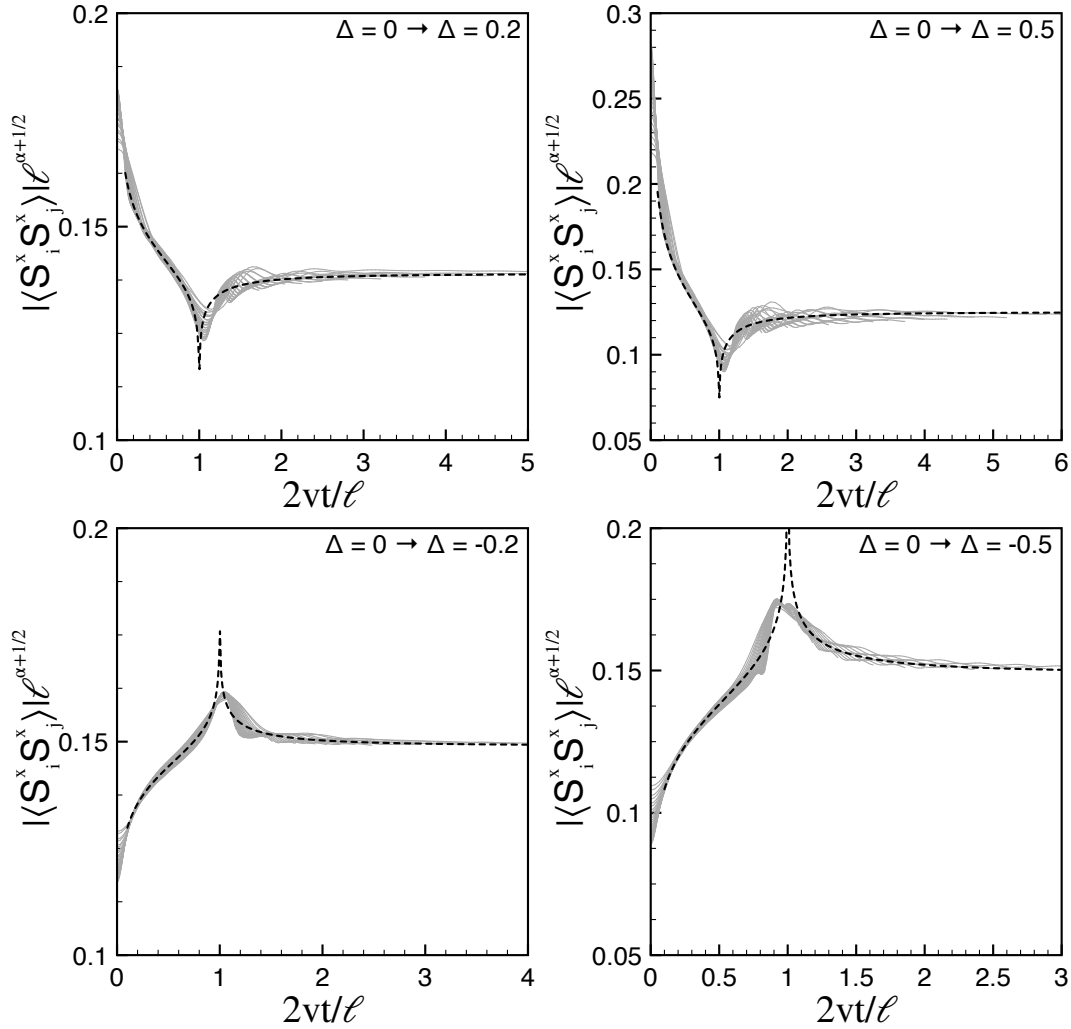
**Figure 3.** Correlation function  $|\langle S_j^x S_{j+l}^x \rangle|$  at time  $t = 20$ , plotted as a function of the lattice distance  $\ell$ , for different interaction strengths  $\Delta$  (different symbols). For comparison we also report the initial correlators (black squares). The dashed lines represent the asymptotic LL predictions  $\sim \ell^{-\alpha-1/2}$ .

for the quenches to  $\Delta = -0.5$  and  $\Delta = -0.2$  (where the propagation velocity is lower) these regions are visible in Fig. 3.

Having established the main qualitative features of the transverse correlator, we now turn to a quantitative analysis of the crossover between the two regimes described by Eq. (13). To that end we perform linear fits of our numerical results to (13) as a function of time for fixed distances  $\ell$ . The parameters  $K$  and  $v$  are fixed by the quench parameter  $\Delta$

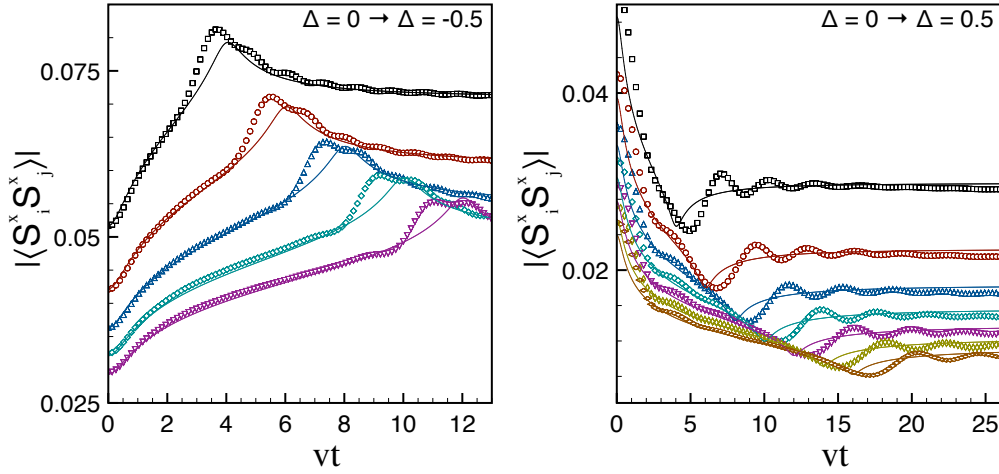
$\Delta$	$\delta A^x$	$\delta A^z$	$\delta B^z$
-0.5	6%	60%	5%
-0.2	2%	10%	5%
0.2	2%	10%	20%
0.5	6%	6%	15%

**Table 2.** Approximate relative deviations of the fitting parameters from the *equilibrium* Luttinger values in Eqs. (9) and (11). The large relative error in  $A^z(\ell)$  for  $\Delta = -0.5$  is due to the fact that equilibrium value is very close to zero (see Fig. 10).



**Figure 4.** Space-time scaling of  $|\langle S_i^x S_j^x \rangle| \ell^{\alpha+1/2}$  with  $\ell = j - i$  for different values of the post-quench interaction strengths  $\Delta$ . After rescaling the correlators, the numerical results (grey lines) for different distances  $\ell$  collapse almost one on top of one another. The dashed black lines is the universal Luttinger liquid prediction.

and reported in Table 1. We repeat the fit procedure for several time-windows  $[1, t_{max}]$  with  $t_{max} = 12, 16, 20$ . The fit parameter  $A^x$  is found to depend only weakly on the choice of  $t_{max}$ , see Figs 1 and 2). We find that the values of  $A^x$  determined in this way are rather close to their “equilibrium” values (9), with deviations of the order of a few percent. In Table 2 we report estimates of the relative difference  $\delta A^x \equiv (A^x(\ell) - A_0^x)/A_0^x$  for all considered quenches, where we perform an average over the values of  $\ell$  which we consider large enough. On the other hand, fixing  $A^x$  to its equilibrium value results in significantly poorer agreement between our numerical results and the Luttinger liquid prediction (13).



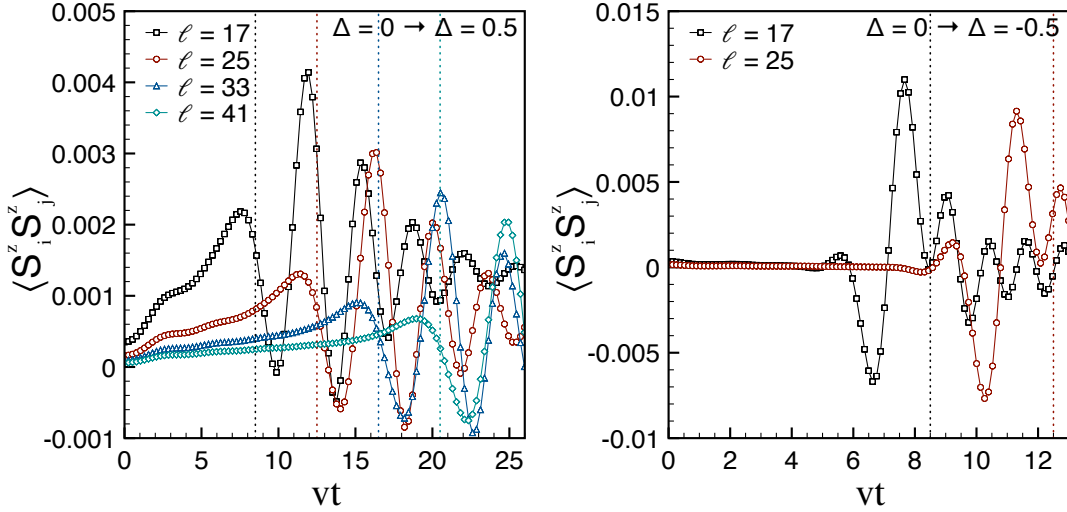
**Figure 5.** Fit of  $|\langle S_i^x S_j^x \rangle|$  with  $\ell = j - i$  to the LL prediction taking into account cutoff effects through Eq. (A.12). The cutoff improves the agreement with the numerical data for short times, but not in the vicinity of the light cone. Left panel:  $\ell = 8, 12, 16, 20, 24$ . Right panel:  $\ell = 8, 12, 16, 20, 24, 28, 32$ .

#### 4.1. Scaling behaviour

A different way of exhibiting the level of agreement between our numerical results and the LL predictions is presented in Fig. 4, which displays the space-time scaling behaviour of the transverse correlation function in terms of the rescaled variable  $2vt/\ell$ . The Luttinger liquid result (13) predicts that correlator for fixed  $\Delta$  but different distances  $\ell$  should collapse on top of the same scaling function, once it has been rescaled by  $\ell^{1/2+\alpha}$ . Of course, the usual restrictions  $\ell, vt, |\ell - 2vt| \gg a$ , cf. section 2.1, continue to apply. Fig. 4 demonstrates that our numerical results indeed show a very nice data collapse for all considered quenches. This is consistent with the good stability of the fitting parameter  $A^x$  shown in Figs 1 and 2. We note that the scaling plots in Fig. 4 are in good agreement with the LL prediction even rather close to the light cone: the smoothed peaks get sharper with increasing  $\ell$  and they get closer and closer to the asymptotic LL results. The good agreement of our numerical results with the LL prediction is rather surprising, given that our fits have been performed with a single free parameter,  $A^x$ , which moreover turns out to be very close to its equilibrium value.

#### 4.2. Cutoff effects

In the previous section we concluded that Luttinger liquid theory provides a very good description of the quench dynamics of the XXZ Hamiltonian as long as  $\ell, vt, |\ell - 2vt| \gg a_0 = 1$ . A natural question is whether retaining an adjustable cutoff parameter in the LL approach could lead to an improved description of the short-time dynamics and the behaviour close to the light cone. The effect of the cutoff in on the transverse spin correlator is given by Eq. (A.12), where  $a$  is proportional but not necessarily equal to  $a_0$ . Using  $a$  as a fit parameter we obtain the results shown in Fig. 5. We find



**Figure 6.**  $\langle S_i^z S_j^z \rangle$  as function of time for several separations  $\ell = j - i$ . Large oscillations appear when the light cone (represented by a vertical line of the same colour as the data) is approached.

that, reassuringly, the amplitude  $A^x$  is left unchanged by the introduction of the cutoff. Inspection of Fig. 5 reveals that fitting the value of the cutoff significantly improves the agreement with the data for short times, but not close to the light cone. The main remaining difference between our numerical results and the LL prediction takes the form of a small oscillatory contribution inside the light cone. This effect goes beyond the LL approximation, but is small in the transverse correlation function. This will no longer be the case for the longitudinal correlation function, to which we turn next.

## 5. Longitudinal spin correlation functions

The analysis of the longitudinal correlation function  $\langle S_j^z S_{j+\ell}^z \rangle$  is significantly more involved than that of  $\langle S_j^x S_{j+\ell}^x \rangle$ . This is because the LL prediction (14) now involves several contributions of similar size, and because there are substantial effects not captured by simple Luttinger liquid theory. The latter are strongest close to the light cone at  $t^* = \ell/2v$  and we now turn to their description.

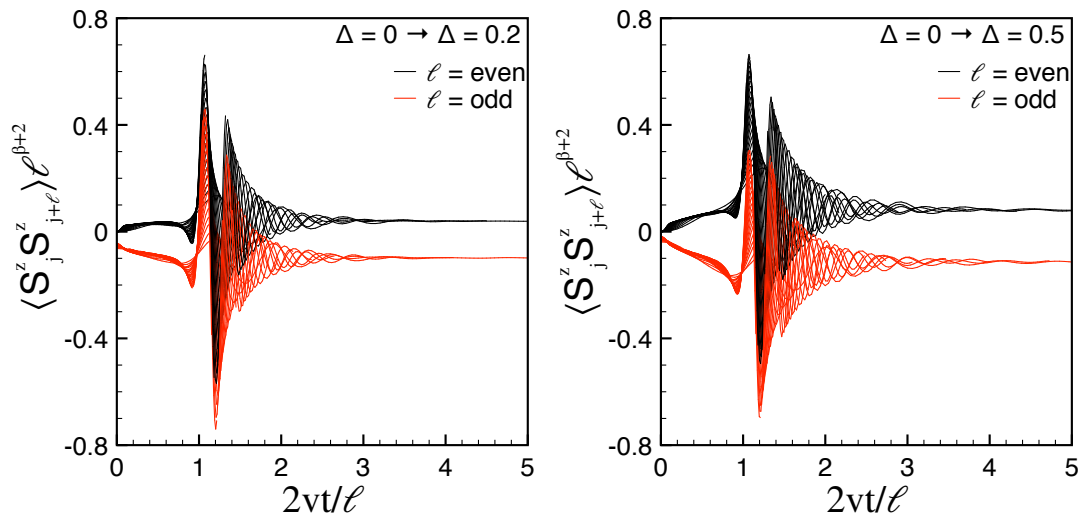
### 5.1. Vicinity of the light cone

In Fig. 6 we show some typical results for quenches to positive (left panel) and negative (right panel) values of  $\Delta$ .

We fix the distance  $\ell$  to be an odd integer in order to account for a strong even/odd effect present in the initial correlations

$$\langle S_j^z S_{j+\ell}^z \rangle \Big|_{t=0} = \frac{(-1)^\ell - 1}{2\pi^2 \ell^2}. \quad (16)$$

We note that this effect is captured by the LL approximation (14), since the smooth and the staggered terms (proportional respectively to  $B^z$  and  $A^z$ ) are very close in

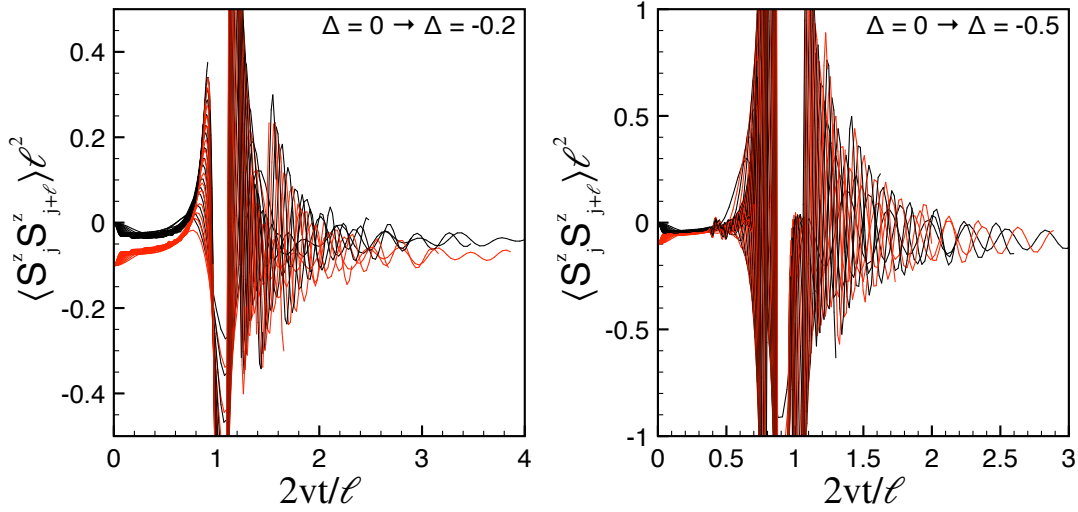


**Figure 7.** Space-time scaling of  $\langle S_j^z S_{j+\ell}^z \rangle$  for  $\Delta > 0$ . The numerical results (black and red lines) for different distances  $\ell$  generate two regular envelopes associated with even and odd  $\ell$  respectively.

magnitude. Inspection of Fig. 6 shows that as the light cone (represented by dashed vertical lines in the figure) is approached for a given  $\ell$ , large oscillations in time ensue. This behaviour is plainly not encoded in the Luttinger liquid approximation and can be understood qualitatively as follows. In the Luttinger liquid approximation the longitudinal spin correlation function exhibits a strong (quadratic) singularity at the light cone. The behaviour in this regime is determined by the high-energy modes, i.e. the ultraviolet part of the spectrum. However, at high energies the Luttinger liquid approximation is no longer expected to provide a good description of the Heisenberg model, as lattice effects (such as saddle points of the spinon dispersion) are important. On the other hand, one may hope that Luttinger liquid approximation is applicable sufficiently far away from the light cone. In order to investigate this possibility it is useful to consider the short-time ( $2vt \leq \ell$ ) and the long-time ( $2vt \geq \ell$ ) behaviour separately.

## 5.2. Space-time scaling behaviour

Before starting the quantitative analysis, we discuss the space-time scaling of this correlator. In the space-time scaling regime with  $\ell, t \rightarrow \infty$  with their ratio fixed, the staggered and the smooth terms scale differently with the latter going like  $\ell^{-2}$  while the former like  $\ell^{-2-\beta}$  with  $\beta = K^2 - 1$ . Given the dependence of  $K$  on  $\Delta$ , cf. Eq. (3), the staggered term is leading for  $\Delta > 0$  while the smooth contribution dominates for  $\Delta < 0$ . We first consider the repulsive regime  $\Delta > 0$ . In Fig. 7 we show a space-time scaling plot of  $\langle S_j^z S_{j+\ell}^z \rangle \ell^{\beta+2}$  as a function of the scaled variable  $2vt/\ell$  for several values of  $\ell$ . The most striking feature of the longitudinal correlations are their pronounced enhancement in the vicinity of the light cone, as compared to their initial values (in absolute terms the correlations are still small due to the factor of  $\ell^{\beta+2}$ ). This is quite



**Figure 8.** Space-time scaling of  $\langle S_j^z S_{j+\ell}^z \rangle$  for  $\Delta < 0$ . We observe fair data collapse at short times, and irregular envelopes at late times.

different from behaviour seen in the transverse spin-spin correlation function.

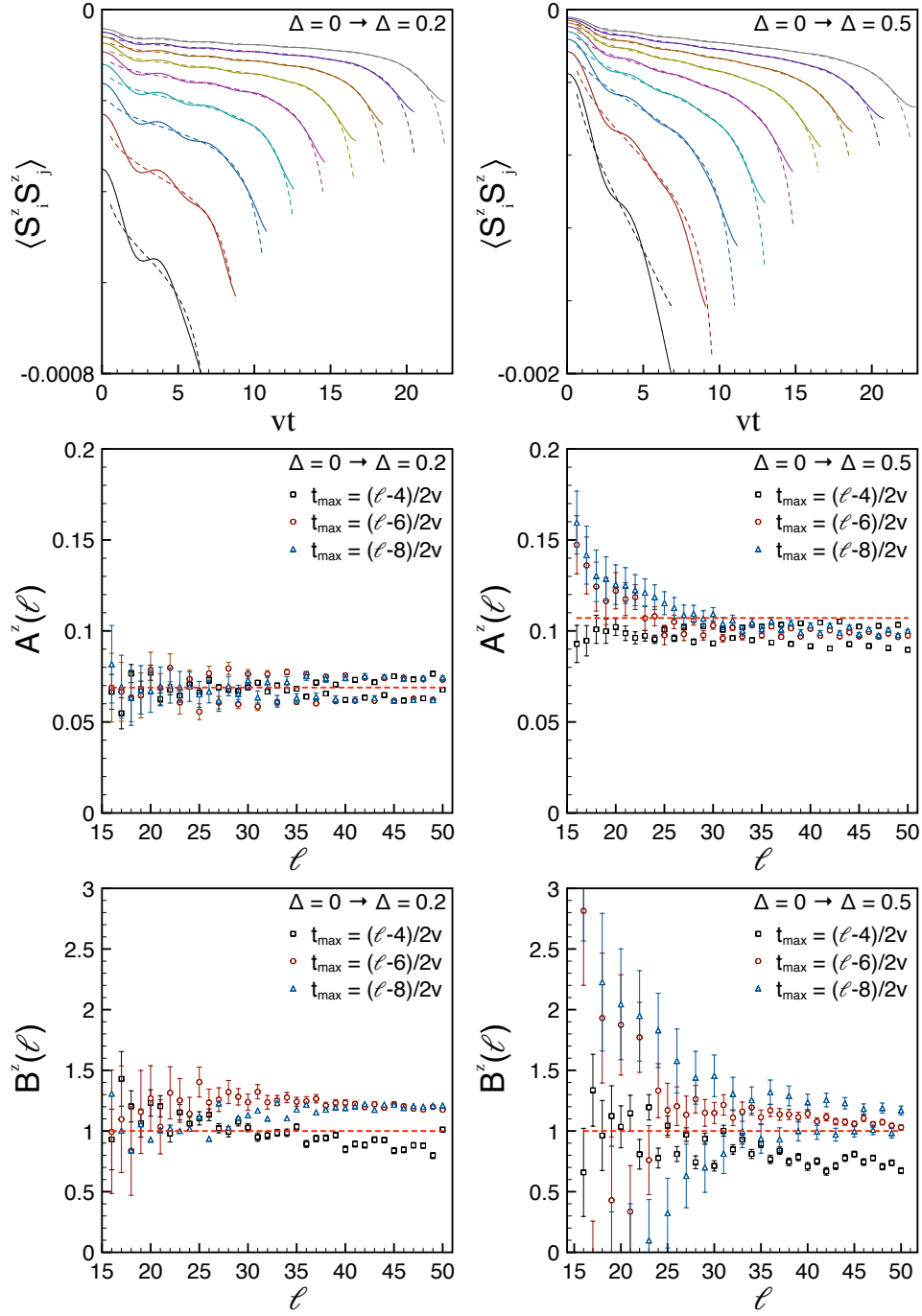
For a given parity of  $\ell$  the numerical results are seen to exhibit a fair data collapse outside the light cone. The absence of symmetry around zero has its origin in the presence of both staggered and smooth contributions to the correlation function. In the vicinity of the light cone as well as in its interior the oscillations described above spoil any data collapse, but the different curves form nice, regular envelopes.

In the attractive regime we consider the scaling behaviour of  $\langle S_j^z S_{j+\ell}^z \rangle \ell^2$ . As is shown in Fig. 8, there again is a very strong enhancement of longitudinal correlations in the vicinity of the light cone. There is a good data collapse only for short times and large oscillations start playing an important role at an early stage of the time evolution (for  $\Delta = -0.5$  the oscillations start at  $2vt/\ell \simeq 0.4$ ). Moreover, we observe that the oscillations are much more irregular than for  $\Delta > 0$ .

### 5.3. Behaviour outside the light cone $2vt \leq \ell$

We now turn to a quantitative analysis of the longitudinal correlations outside the light cone, i.e. at short times  $t \leq \ell/2v$ . The time dependence of  $\langle S_j^z S_{j+\ell}^z \rangle$  is shown in the top two panels of Fig. 9 for quenches to  $\Delta = 0.2$ ,  $\Delta = 0.5$  and in Fig. 10 for quenches to  $\Delta = -0.2$ ,  $\Delta = -0.5$ . The numerical results are compared to best fits to the Luttinger liquid prediction (14), which are shown as dashed lines. The agreement is seen to be satisfactory. The fitted values for the amplitudes  $A^z$  and  $B^z$  are shown in the middle and bottoms panels of Figs 9 and 10 respectively.

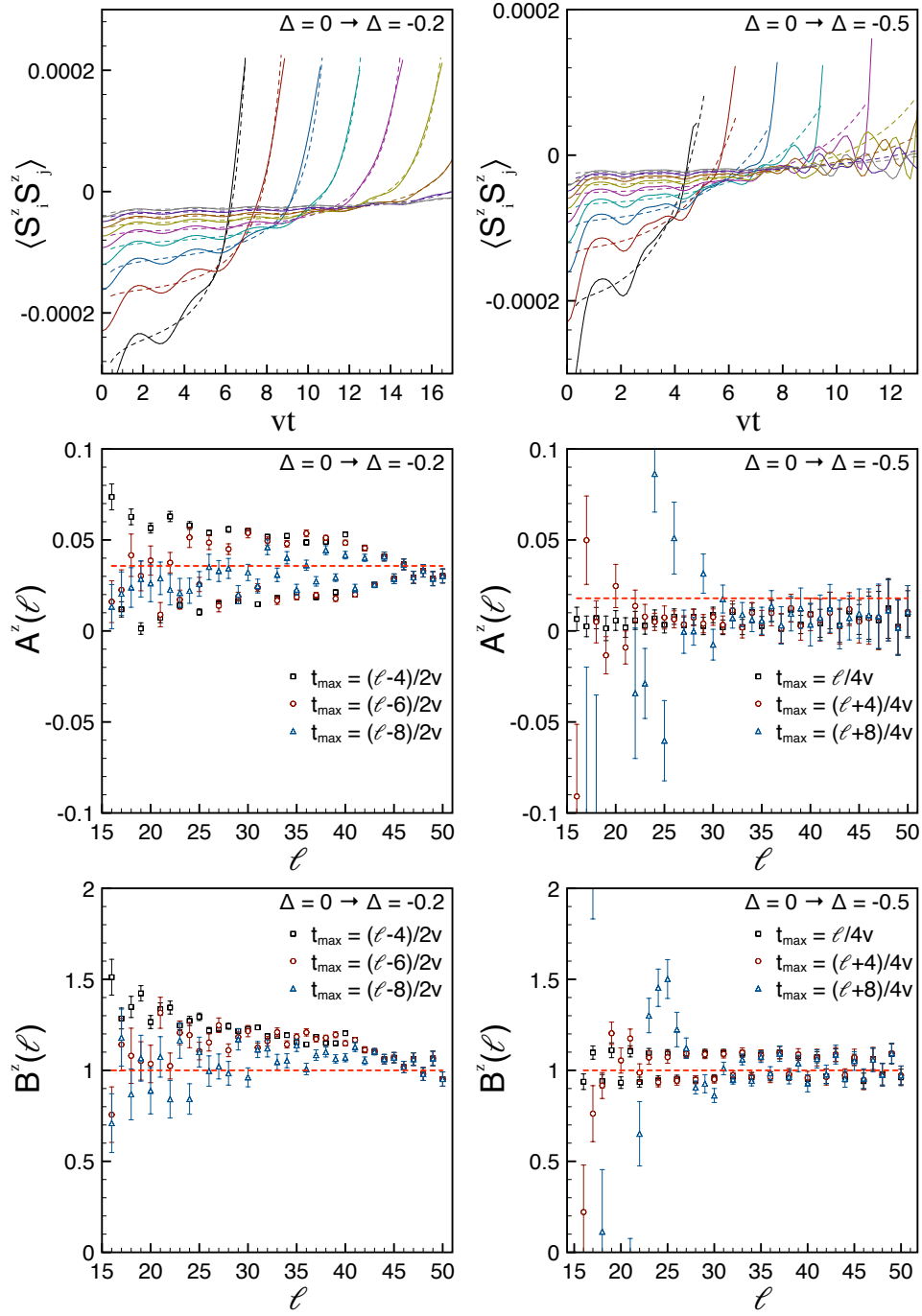
Some comments on our fit procedure are in order. To avoid the strong light-cone singularity in the LL approximation at  $\ell = 2vt$ , we performed fits in the time window  $[1, (\ell - \delta\ell)/2v]$  for several values of  $\delta\ell$ . These are shown in the bottom four panels of Figs 9 and 10). We observe a quite satisfactory stability of the coefficients  $A^z$  and



**Figure 9.** (Top) Short time regime of  $\langle S_i^z S_j^z \rangle$ , for two different interaction strengths in the repulsive regime  $\Delta > 0$  and distances  $\ell = j - i = 17, 21, 25, \dots, 49$ . The numerical data are shown as full lines, while dashed lines represent best fits with the LL approximation (fitting with  $t_{\max} = (\ell - 6)/2v$ ). (Centre/Bottom) Amplitudes  $A^z$  and  $B^z$  obtained from best fits with different values of  $t_{\max}$ . The dashed red lines are the equilibrium values of the amplitude, i.e.  $A_0^z = 1$ ,  $A_1^z = 0.0689$  (for  $\Delta = 0.2$ ) and  $A_1^z = 0.1071$  (for  $\Delta = 0.5$ ).

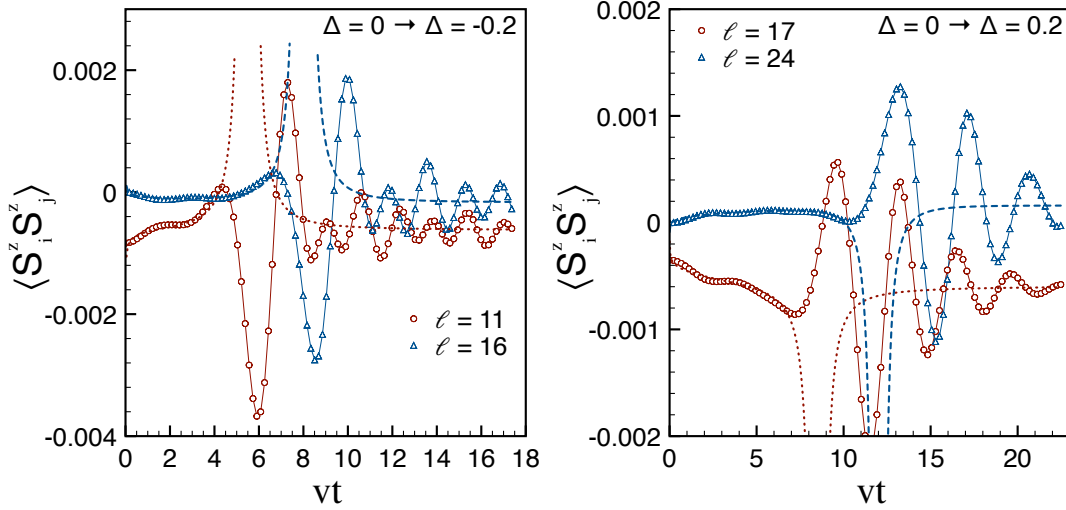
$B^z$  against changing the time interval over which the fits are performed. This stability improves with increasing  $\ell$ , i.e. gets better in the regime where the LL approximation





**Figure 10.** (Top) Short time regime of  $\langle S_i^z S_j^z \rangle$  for two interaction strengths in the attractive regime  $\Delta < 0$  and distances  $\ell = j - i = 17, 21, 25, \dots, 49$ . The numerical data are shown as full lines, while dashed lines represent best fits with the LL approximation (fitting with  $t_{\max} = (\ell - 6)/2v$  for  $\Delta = -0.2$  and with  $t_{\max} = \ell/4v$  for  $\Delta = -0.5$ ). (Centre/Bottom) Amplitudes  $A^z$  and  $B^z$  obtained from fits with different  $t_{\max}$ . The dashed red lines are the equilibrium values of the amplitude, i.e.  $A_0^z = 1$ ,  $A_1^z = 0.0357$  (for  $\Delta = -0.2$ ) and  $A_1^z = 0.0179$  (for  $\Delta = -0.5$ ).

is expected to work best. We observe a small even/odd effect in the fitted values of  $A^z$  and  $B^z$ , which we attribute to subleading terms within Luttinger liquid theory. The



**Figure 11.** Numerical results for  $\langle S_i^z S_j^z \rangle$  compared (for several distances  $\ell = j - i$ ) to the LL approximation with amplitudes fixed by fitting only short times. While the oscillatory behaviour is not captured by the LL approximation, it appears to correctly account for the decay of correlations in the accessible time window.

fitted values of both  $A^z$  and  $B^z$  are approximately the same as in equilibrium. In Table 2 we report some rough estimates of the maximal relative differences.

#### 5.4. LL approximation close to the light cone

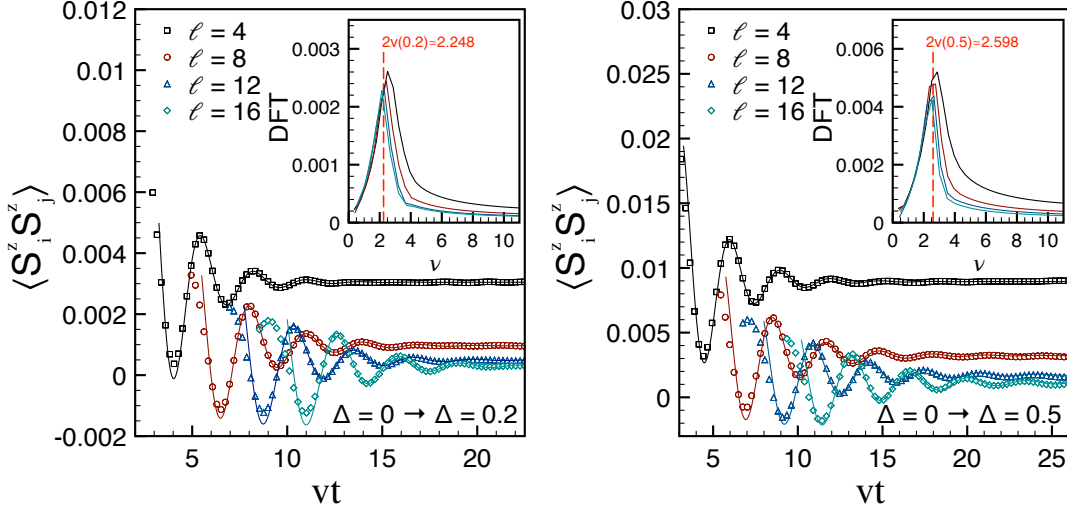
A natural question is to what extent the LL approximation, with amplitudes  $A^z$ ,  $B^z$  determined above, continues to describe the correlation function close to, and inside, the light cone. In Fig. 11 we present results of such comparisons for two different quenches. The correlation function is seen to exhibit decaying oscillatory behaviour inside the light cone. It is evident that the LL approximation fails to capture the strong oscillations. However, in all cases the oscillations appear to be centered around the LL approximation and to decay towards the latter.

#### 5.5. Physics beyond the LL approximation inside the light cone

We have seen that inside the light cone the longitudinal correlation function displays strong oscillatory behaviour, that is not captured by the LL approximation. We now turn to a parameterization of this effect. We have found that the numerical data is well described by the functional form

$$\langle S_j^z S_{j+\ell}^z \rangle \approx B + C \exp(-Dt) \sin(2vt + \phi). \quad (17)$$

Here  $v$  is equal to the ground state velocity of sound and  $\{B, C, D, \phi\}$  are constants that are used to obtain best fits of the numerical results to (17). We use the time window  $[(\ell + \delta\ell)/(2v), 20]$  and then vary  $\delta\ell$  in order to optimize the fit. As is shown in Fig. 12, the quality of these fits is quite good.



**Figure 12.** Large-time behaviour of  $\langle S_i^z S_j^z \rangle$  with  $\ell = j - i$ . The data (symbols) are compared with the best fit of Eq. (17). In the inset we report the DFT of  $\langle S_i^z S_j^z \rangle$ , which displays a peak close to  $\nu_0 = 2v$ .

In order to quantify how well the oscillatory behaviour can be described in terms of a single frequency  $\nu_0 = 2v$  we have carried out a Fourier analysis of our numerical data. Discrete Fourier transforms (DFT) of  $\langle S_i^z S_j^z \rangle$  are shown in the insets of Fig. 12. We see that the oscillation frequencies form a narrow band peaked around  $\nu_0 = 2v$ . Moreover, as the distance  $\ell = j - i$  increases, the frequency distribution narrows and its centre approaches  $\nu_0$ . We also performed fits of our numerical results to (17) treating  $v$  as a fitting parameter. As expected we found that the best value of  $v$  is then extremely close to the sound velocity.

For quenches to  $\Delta < 0$ , the accessible time window  $t \in [0, 20]$  is too small to assess the applicability of (17). The reason is that the sound velocity  $v$  decreases rapidly with decreasing  $\Delta$ , cf. Table 1. This in turn results in a small oscillation frequency in (17), and concomitantly few oscillations inside the light cone at times  $t < 20$ . At the same time our numerical computations are still limited by the growth of entanglement entropy, which exhibits only a weak dependence on the sign of  $\Delta$ . This precludes us from exploring larger time windows.

The fact that the oscillation frequency equals  $\nu = 2v$  gives us an indication of which processes may underlie this behaviour. According to the approach proposed in Ref. [9], the time evolution of the spin-spin correlation function for a quantum quench from the initial state  $|\Psi_0\rangle$  is given by

$$\langle S_j^z S_{j+\ell}^z \rangle = \lim_{N \rightarrow \infty} \text{Re} \sum_{|\psi_n\rangle} e^{\mathcal{E}_{\Phi_s}^* - \mathcal{E}_{\psi_n}^* + it(\omega_{\psi_n} - \omega_{\Phi_s})} \langle \psi_n | S_j^z S_{j+\ell}^z | \Phi_s \rangle. \quad (18)$$

Here  $|\psi_n\rangle$  are eigenstates of the Hamiltonian and  $\omega_{\psi_n}$  the corresponding energies,  $|\Phi_s\rangle$  is a particular simultaneous eigenstates of local conservation laws of the spin-1/2 XXZ chain that describes the steady state,  $\omega_{\Phi_s}$  its energy, and  $\mathcal{E}_{\psi} \equiv -\ln\langle\psi|\Psi_0\rangle$ . It was argued in Ref. [9] that only states with finite energy differences relative to  $\omega_{\Phi_s}$  contribute to

(18). Our numerical analysis suggests that the oscillatory behaviour inside the light cone is well characterized by a single frequency  $\nu_0 = 2v$ . In (18) this corresponds to energy eigenstates  $|\psi_n\rangle$  with

$$\omega_{\psi_n} - \omega_{\Phi_s} = \pm 2v. \quad (19)$$

The spectrum of the XXZ chain in the vicinity of the representative state  $|\Phi_s\rangle$  can be calculated [94] by a generalized thermodynamic Bethe Ansatz [95]. The most important excitations are particle-hole excitations with energy

$$\omega_\alpha(k^p, h^h) = \epsilon_\alpha(k^p) - \epsilon_\alpha(k^h), \quad (20)$$

where the index  $\alpha$  runs over the different types (“strings”) of allowed excitations at anisotropy  $\Delta$  and  $\epsilon_\alpha(k)$  is their dressed energy. We conjecture (cf. Ref. [94]) that for the quenches considered here, the bandwidth of  $\epsilon_1(k)$  is approximately equal to that of the equilibrium spinon dispersion  $\epsilon_s(k)$ , and that the extrema of  $\epsilon_1(k)$  occur at  $k = 0, \pi/2, \pi$  (we are working at zero magnetization). For the zero temperature equilibrium spinon dispersion we have [90]

$$|\epsilon_s(\pi/2) - \epsilon_s(0)| = 2v. \quad (21)$$

These considerations suggest that the oscillatory behaviour inside the light cone originates in particle-hole excitations above the representative state connecting saddle points of  $\epsilon_1(k)$  at  $k = \pi/2$  and  $k = 0$ . These are high-energy degrees of freedom manifestly beyond the applicability of the Luttinger liquid approximation.

### 5.6. Stationary behaviour

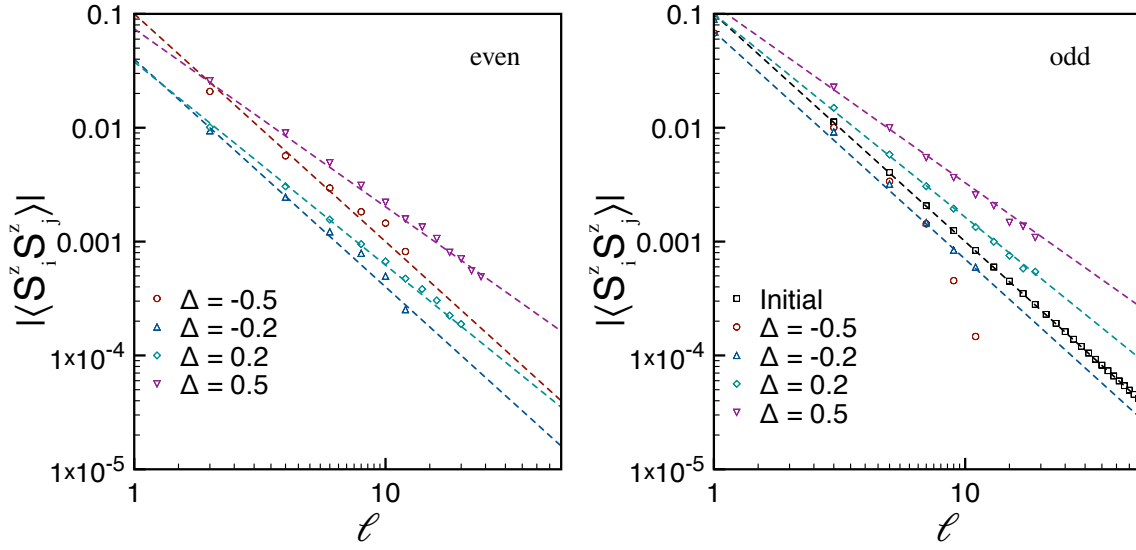
Finally, we turn to the late time behaviour of the longitudinal correlations. The latest time accessible by our numerical computations is  $t = 20$ . In Fig. 13 we display the dependence of  $|\langle S_j^z S_{j+\ell}^z \rangle|$  on the separation  $\ell$  at that time. As the correlator displays even/odd effects due to the presence of both smooth and staggered contributions we show the results for even and odd values of  $\ell$  in separate graphs. The Luttinger liquid approximation (14) predicts that at late times

$$\lim_{t \rightarrow \infty} |\langle S_j^z S_{j+\ell}^z \rangle| \sim \begin{cases} \ell^{-2} & \text{for } \Delta < 0, \\ \ell^{-\beta-2} & \text{for } \Delta > 0. \end{cases} \quad (22)$$

where the exponent  $\beta$  is given in Table 1. Although our numerical results at  $t = 20$  exhibit small oscillations (probably non-stationary), they agree well with the asymptotic LL prediction for all values of  $\Delta$  except for  $\Delta = -0.5$  and odd  $\ell$ . This disagreement is not surprising, because for this value of  $\Delta$  the velocity  $v$  is small and the correlations at  $t = 20$  are therefore not yet stationary for the largest shown values of  $\ell$ .

## 6. Conclusions

We have considered interaction quenches from the ground state of the XX model to the spin-1/2 Heisenberg XXZ chain (1) in the quantum critical regime. In equilibrium



**Figure 13.** Longitudinal correlation function  $\langle S_i^z S_j^z \rangle$  at time  $t = 20$  for several different interaction strengths  $\Delta$  as a function of the distance  $\ell = j - i$ , which is taken to be even in the left panel and odd in the right one. The dashed lines are the asymptotic LL predictions (22). In the right panel the initial ( $t = 0$ ) correlations are shown as back squares. The initial correlations for even  $\ell$  are equal to zero.

the low-energy physics of the post-quench Hamiltonian is described by Luttinger liquid theory, and we have investigated to that extent this description applies to dynamics out of equilibrium. We have computed two-point correlation functions of quantum spins in the XXZ chain and compared our results to predictions obtained by means of a Luttinger liquid approximation. Our work extends previous studies, which focused on other observables [65, 66]. Our main results are as follows. We found that on the time scales accessible to us the transverse correlator  $\langle S_j^x S_{j+\ell}^x \rangle$  is well described by LL theory. Small deviations are seen in the vicinity of the light cone, i.e.  $2vt \approx \ell$ . The good agreement between the LL approximation and the numerical results is surprising and surpasses our most optimistic expectations. Conversely, LL theory fails to give a good description of the longitudinal two-point function  $\langle S_j^z S_{j+\ell}^z \rangle$  except at rather short times. This implies that the longitudinal correlations are more strongly affected by corrections to LL theory.

It is tempting to speculate that this difference in the applicability of the LL approximation could be related to the locality properties of the observables  $S_j^{x,z}$  with respect to the elementary excitations that spread the correlations. While  $S_j^z$  is local in this sense,  $S_j^x$  is not. This can be seen by noting that the elementary excitations are dressed versions of the Jordan-Wigner fermions, and the expression of  $S_j^x$  in terms of the latter involves a string operator. It has been previously noted [56, 51] that local and non-local operators exhibit qualitatively different quench dynamics in particular examples. If our observations are indeed related to locality properties of observables, one would expect analogous behaviour in other models, whose low-energy equilibrium properties are described by Luttinger liquids. One such example is that of interaction quenches

in the Lieb-Liniger model. Here the field correlator (one particle density matrix) would then be well described by the LL approximation, while the density-density correlation would not. This is an opportune conjecture because in the Lieb-Liniger model interaction quenches (between two finite values of the interaction) are experimentally feasible, but there is still no analytic or numerical method to tackle its non-equilibrium dynamics.

Finally, our results are relevant to the physics of prethermalization [96, 97, 98, 99, 100, 101, 102, 103] in models with weak integrability breaking. A prethermalized regime has been observed in experiments [97] and described using Luttinger liquid methods. Our work shows that on intermediate time scales genuine high-energy effects need to be taken into account as well as “irrelevant” perturbations to the Luttinger liquid Hamiltonian [73].

## Acknowledgments

All authors acknowledge the financial support by the ERC under Starting Grant 279391 EDEQS. The work of FHL was supported by the EPSRC under grants EP/J014885/1 and EP/I032487/1.

## Appendix A. Interaction quench in a Luttinger liquid

Our starting point is the Luttinger liquid Hamiltonian

$$\mathcal{H}(\Delta) = \frac{v}{2} \int dx \left[ K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 \right]. \quad (\text{A.1})$$

After carrying out a canonical transformation

$$\tilde{\phi} = \tilde{\varphi} + \tilde{\bar{\varphi}} = \frac{\phi}{\sqrt{K}}, \quad \tilde{\theta} = \tilde{\varphi} - \tilde{\bar{\varphi}} = \theta\sqrt{K}, \quad (\text{A.2})$$

the time evolution (at  $t > 0$ ) of the chiral fields is described by the mode expansion

$$\begin{aligned} \tilde{\varphi}(t, x) &= \tilde{Q} + \frac{\tilde{P}}{2L}(vt - x) + \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[ \tilde{\beta}_n e^{ik_n(x-vt)} + \text{h.c.} \right], \\ \tilde{\bar{\varphi}}(t, x) &= \tilde{\bar{Q}} + \frac{\tilde{\bar{P}}}{2L}(vt + x) + \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[ \tilde{\beta}_{-n} e^{-ik_n(x+vt)} + \text{h.c.} \right]. \end{aligned} \quad (\text{A.3})$$

Here we consider a ring of circumference  $L$ ,  $k_n = 2\pi n/L$ , and the various mode operators fulfil commutation relations

$$[Q, P] = i = [\bar{Q}, \bar{P}], \quad [\tilde{\beta}_n, \tilde{\beta}_m^\dagger] = \delta_{n,m}. \quad (\text{A.4})$$

At  $t = 0$  our system is prepared in the ground state of (A.1) with  $K = 1$  and  $v = Ja_0$ . This state is conveniently constructed through a mode expansion of  $\mathcal{H}(\Delta = 0)$ , which reads

$$\begin{aligned} \varphi(t = 0, x) &= Q - \frac{P}{2L}x + \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[ \beta_n e^{ik_n x} + \text{h.c.} \right], \\ \bar{\varphi}(t = 0, x) &= \bar{Q} + \frac{\bar{P}}{2L}x + \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[ \beta_{-n} e^{-ik_n x} + \text{h.c.} \right]. \end{aligned} \quad (\text{A.5})$$

The ground state at  $t = 0$  is then defined through the property

$$\beta_n|0\rangle_0 = 0 = P|0\rangle_0 = \bar{P}|0\rangle_0. \quad (\text{A.6})$$

Next, we use the fact that the mode operators in (A.5) and (A.3) are related through the transformation (A.2), which implies that

$$\tilde{\beta}_n = \frac{1}{2} \left[ \frac{1}{\sqrt{K}} + \sqrt{K} \right] \beta_n + \frac{1}{2} \left[ \frac{1}{\sqrt{K}} - \sqrt{K} \right] \beta_{-n}^\dagger. \quad (\text{A.7})$$

The zero mode operators are related by analogous expressions. We are now in a position to work out two point functions of local operators after our interaction quench. Let us consider

$$F(t, x) = {}_0\langle 0 | \exp \left( -i\sqrt{\frac{\pi}{K}} \tilde{\theta}(t, x) \right) \exp \left( i\sqrt{\frac{\pi}{K}} \tilde{\theta}(t, 0) \right) | 0 \rangle_0. \quad (\text{A.8})$$

This can be calculated using the mode expansions (A.3), and then expressing the mode operators using (A.7). The resulting expression is then evaluated using

$${}_0\langle 0 | e^{\gamma\beta_n + \delta b_n^\dagger} | 0 \rangle_0 = e^{\gamma\delta/2}, \quad (\text{A.9})$$

which is a simple consequence of (A.6). As we are ultimately interested in the thermodynamic limit we ignore the contributions arising from the zero modes. We find

$$\begin{aligned} \ln(F(t, x)) = & - \sum_{n=1}^{\infty} \frac{1}{n} \left[ \frac{1 + K^2}{8K^2} (2 - e^{ik_n x} - e^{-ik_n x}) \right. \\ & \left. + \frac{1 - K^2}{16K^2} (e^{ik_n(x+2vt)} + e^{ik_n(x-2vt)} - 2e^{2ik_n vt} + \text{c.c.}) \right]. \end{aligned} \quad (\text{A.10})$$

In order to proceed, we regulate the remaining sum by multiplying the summand by a factor  $e^{-2\pi\epsilon n}$ . For  $\epsilon > 0$  the sum then exists and is evaluated using

$$\ln(1 - z) = - \sum_{n=1}^{\infty} \frac{z^n}{n}. \quad (\text{A.11})$$

After a short calculation we arrive at

$$F(t, x) = \left[ 1 + \frac{x^2}{a^2} \right]^{-\frac{1+K^2}{8K^2}} \left[ \frac{[(x+2vt)^2 + a^2][(x-2vt)^2 + a^2]}{[(2vt)^2 + a^2]^2} \right]^{\frac{1-K^2}{16K^2}}, \quad (\text{A.12})$$

where  $a = \epsilon L$  is a short distance cutoff. In order to make contact with the XXZ spin chain, we need to set  $a$  proportional to the lattice spacing  $a_0$  (because the cutoffs employed in the initial and final Hamiltonian are not the same), use that  $|x|, |x \pm vt|, vt \gg a_0$ , and finally employ (5). This gives the result (13) quoted in the main text. All other correlators are evaluated in an analogous fashion.

## References

- [1] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. **83**, 863 (2011).
- [2] J. Eisert, M. Friesdorf, and C. Gogolin, Nature Phys. **11**, 124 (2015);  
C. Gogolin and J. Eisert, arXiv:1503.07538.

- [3] J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991);  
M. Srednicki, Phys. Rev. E **50**, 888 (1994).
- [4] M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008);  
M. Rigol, Phys. Rev. Lett. **103**, 100403 (2009);  
M. Rigol, Phys. Rev. A **80**, 053607 (2009).
- [5] G. Biroli, C. Kollath, and A. Laeuchli, Phys. Rev. Lett. **105**, 250401 (2010);  
M. C. Banuls, J. I. Cirac, and M. B. Hastings, Phys. Rev. Lett. **106**, 050405 (2011);  
M. Rigol and M. Fitzpatrick, Phys. Rev. A **84**, 033640 (2011);  
K. He and M. Rigol, Phys. Rev. A **85**, 063609 (2012);  
G. P. Brandino, A. De Luca, R. M. Konik, and G. Mussardo, Phys. Rev. B **85**, 214435 (2012);  
M. Rigol and M. Srednicki, Phys. Rev. Lett. **108**, 110601 (2012);  
J. Sirker, N.P. Konstantinidis, and N. Sedlmayr, Phys. Rev. A **89**, 042104 (2014).
- [6] T. Kinoshita, T. Wenger, D. S. Weiss, Nature **440**, 900 (2006).
- [7] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys. Rev. Lett. **98**, 50405 (2007).
- [8] M. Rigol and M. Srednicki, Phys. Rev. Lett. **108**, 110601 (2012).
- [9] J.-S. Caux and F. H. L. Essler, Phys. Rev. Lett. **110**, 257203 (2013).
- [10] P. Calabrese, F. H. L. Essler, and M. Fagotti, J. Stat. Mech. P07022 (2012).
- [11] M. Fagotti and F. H. L. Essler, Phys. Rev. B **87**, 245107 (2013).
- [12] D. Fioretto and G. Mussardo, New J. Phys. **12**, 055015 (2010);  
S. Sotiriadis, D. Fioretto, and G. Mussardo, J. Stat. Mech. P02017 (2012).
- [13] V. Gritsev, T. Rostunov, and E. Demler, J. Stat. Mech. P05012 (2010).
- [14] D. Iyer and N. Andrei, Phys. Rev. Lett. **109**, 115304 (2012);  
D. Iyer, H. Guan, and N. Andrei, Phys. Rev. A **87**, 053628 (2013).
- [15] P. Calabrese and J. Cardy, Phys. Rev. Lett. **96**, 136801 (2006);  
P. Calabrese and J. Cardy, J. Stat. Mech. P06008 (2007).
- [16] B. Bertini, D. Schuricht, and F. H. L. Essler, J. Stat. Mech. (2014) P10035.
- [17] H. Buljan, R. Pezer, and T. Gasenzer, Phys. Rev. Lett. **100**, 080406 (2008).
- [18] J.-S. Caux and R. M. Konik, Phys. Rev. Lett. **109**, 175301 (2012).
- [19] J. De Nardis, B. Wouters, M. Brockmann, and J.-S. Caux, Phys. Rev. A **89**, 033601 (2014).
- [20] G. Mussardo, Phys. Rev. Lett. **111**, 100401 (2013).
- [21] G. Goldstein and N. Andrei, arXiv:1505.02585.
- [22] M. Fagotti and F. H. L. Essler, J. Stat. Mech. P07012 (2013).
- [23] M. Fagotti, M. Collura, F. H. L. Essler, and P. Calabrese, Phys. Rev. B **89**, 125101 (2014).
- [24] B. Wouters, M. Brockmann, J. De Nardis, D. Fioretto, M. Rigol, and J.-S. Caux, Phys. Rev. Lett. **113**, 117202 (2014);  
M. Brockmann, B. Wouters, D. Fioretto, J. De Nardis, R. Vlijm, and J.-S. Caux, J. Stat. Mech. (2014) P12009.
- [25] B. Pozsgay, M. Mestyán, W. A. Werner, M. Kormos, G. Zaránd, and G. Takács, Phys. Rev. Lett. **113** (2014) 117203;  
M. Mestyán, B. Pozsgay, G. Takacs, and M.A. Werner, J. Stat. Mech. (2015) P04001.
- [26] W. Liu and N. Andrei, Phys. Rev. Lett. **112**, 257204 (2014).
- [27] A. Faribault, P. Calabrese, and J.-S. Caux, J. Stat. Mech. P03018 (2009);  
A. Faribault, P. Calabrese, and J.-S. Caux, J. Math. Phys. **50**, 095212 (2009).
- [28] S. Sotiriadis, G. Takacs, and G. Mussardo, Phys. Lett. B **734** (2014).
- [29] P. Calabrese and P. Le Doussal, J. Stat. Mech. P05004 (2014).
- [30] M. Kormos, A. Shashi, Y.-Z. Chou, J.-S. Caux, and A. Imambekov, Phys. Rev. B **88**, 205131 (2013);  
B. Pozsgay J. Stat. Mech. (2014) P10045.
- [31] M. Kormos, M. Collura, and P. Calabrese, Phys. Rev. A **89**, 013609 (2014);  
M. Collura, M. Kormos, and P. Calabrese, J. Stat. Mech. P01009 (2014);  
P. P. Mazza, M. Collura, M. Kormos, and P. Calabrese, J. Stat. Mech. P11016 (2014).



- [32] J. De Nardis and J.-S. Caux, J. Stat. Mech. (2014) P12012.
- [33] J. De Nardis, L. Piroli, and J.-S. Caux, arXiv:1505.03080.
- [34] B. Pozsgay, J. Stat. Mech. P10028 (2013);  
B. Pozsgay, J. Stat. Mech. P06011 (2014).
- [35] M. Brockmann, J. De Nardis, B. Wouters, and J.-S. Caux, J. Phys. A **47**, 145003 (2014).
- [36] M. Brockmann, J. De Nardis, B. Wouters, and J.-S. Caux, J. Phys. A **47**, 345003 (2014).
- [37] M. Brockmann, J. Stat. Mech. P05006 (2014).
- [38] L. Piroli and P. Calabrese, J. Phys. A **47**, 385003 (2014).
- [39] G. Goldstein and N. Andrei, Phys. Rev. A **90**, 043625 (2014);  
B. Pozsgay, arXiv:1406.4613.
- [40] R. G. Pereira, V. Pasquier, J. Sirker, and I. Affleck, J. Stat. Mech. (2014) P09037.
- [41] M. Mierzejewski, P. Prelovsek, and T. Prosen, Phys. Rev. Lett. **114**, 140601 (2015).
- [42] F.H.L. Essler, G. Mussardo, M. Panfil, Phys. Rev. A **91**, 051602 (2015).
- [43] E. Ilievski, M. Medenjak, and T. Prosen, arXiv:1506.05049.
- [44] M. Fagotti, arXiv:1408.1950.
- [45] E. Ilievski, J. De Nardis, B. Wouters, J.-S. Caux, F. H. L. Essler, and T. Prosen, arxiv:1507.02993.
- [46] M. A. Cazalilla, Phys. Rev. Lett. **97**, 156403 (2006).
- [47] A. Iucci and M. A. Cazalilla, Phys. Rev. A **80**, 063619 (2009).
- [48] A. Iucci and M. A. Cazalilla New J. Phys. **12**, 055019 (2010).
- [49] M. A. Cazalilla, A. Iucci, and M.-C. Chung, Phys. Rev. E **85**, 011133 (2012).
- [50] T. Barthel and U. Schollwöck, Phys. Rev. Lett. **100**, 100601 (2008);  
M. Cramer, C. M. Dawson, J. Eisert, and T. J. Osborne, Phys. Rev. Lett. **100**, 030602 (2008);  
M. Cramer and J. Eisert, New J. Phys. **12**, 055020 (2010);  
S. Sotiriadis and P. Calabrese, J. Stat. Mech. P07024 (2014).
- [51] P. Calabrese, F.H.L. Essler, and M. Fagotti, Phys. Rev. Lett. **106**, 227203 (2011);  
P. Calabrese, F. H. L. Essler, and M. Fagotti, J. Stat. Mech. P07016 (2012);  
F. H. L. Essler, S. Evangelisti, and M. Fagotti, Phys. Rev. Lett. **109**, 247206 (2012).
- [52] S. Sotiriadis, P. Calabrese, and J. Cardy, EPL **87**, 20002 (2009).
- [53] J. Mossel and J.-S. Caux, New J. Phys. **14**, 075006 (2012).
- [54] M. Fagotti and P. Calabrese, Phys. Rev. A **78**, 010306 (2008).
- [55] A. Silva, Phys. Rev. Lett. **101**, 120603 (2008).
- [56] D. Rossini, A. Silva, G. Mussardo, and G. Santoro, Phys. Rev. Lett. **102**, 127204 (2009);  
D. Rossini, S. Suzuki, G. Mussardo, G. Santoro, and A. Silva, Phys. Rev. B **82**, 144302 (2010).
- [57] F. Igłó and H. Rieger, Phys. Rev. Lett. **85**, 3233 (2000);  
F. Igłó and H. Rieger, Phys. Rev. Lett. **106**, 035701 (2011);  
H. Rieger and F. Igłó, Phys. Rev. B **84**, 165117 (2011).
- [58] L. Foini, L. F. Cugliandolo, and A. Gambassi, Phys. Rev. B **84**, 212404 (2011);  
L. Foini, L. F. Cugliandolo, and A. Gambassi, J. Stat. Mech. P09011 (2012).
- [59] D. Schuricht and F. H. L. Essler, J. Stat. Mech. P04017 (2012).
- [60] M. Collura, S. Sotiriadis, and P. Calabrese, Phys. Rev. Lett. **110**, 245301 (2013);  
M. Collura, S. Sotiriadis and P. Calabrese, J. Stat. Mech. P09025 (2013).
- [61] M. Fagotti, Rev. B **87**, 165106 (2013);  
L. Bucciandini, M. Kormos, and P. Calabrese, J. Phys. A **47**, 175002 (2014);  
M. Kormos, L. Bucciandini, and P. Calabrese, EPL **107**, 40002 (2014).
- [62] P. Barmettler, M. Punk, V. Gritsev, E. Demler, and E. Altman, Phys. Rev. Lett. **102**, 130603 (2009);  
P. Barmettler, M. Punk, V. Gritsev, E. Demler, and E. Altman, New J. Phys. **12** 055017 (2010).
- [63] M. Fagotti, J. Stat. Mech. P03016 (2014).
- [64] G. De Chiara, S. Montangero, P. Calabrese, and R. Fazio, J. Stat. Mech. P03001 (2006).
- [65] C. Karrasch, J. Rentrop, D. Schuricht, V. Meden, Phys. Rev. Lett. **109**, 126406 (2012).
- [66] E. Coira, F. Becca, and A. Parola, Eur. Phys. J. B **86**, 55 (2013).

- [67] A. Luther and I. Peschel, Phys. Rev. B **12**, 3908 (1975);  
F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1982).
- [68] I. Affleck, in *Fields, Strings and Critical Phenomena*, eds E. Brézin and J. Zinn-Justin, (Elsevier, Amsterdam, 1989);  
A. O. Gogolin, A. A. Nersisyan, and A. M. Tsvelik, *Bosonization in Strongly Correlated Systems* (Cambridge University Press, 1999);  
T. Giamarchi, *Quantum physics in One Dimension*, Clarendon Press, Oxford, (2004);  
J. von Delft and H. Schoeller, *Bosonization for Beginners – Referredmionization for Experts*, cond-mat/9805275;  
J. Sirker, Int. J. Mod. Phys. B **26**, 1244009 (2012).
- [69] S. Lukyanov, Nucl. Phys. B **522**, 533 (1998);  
S. Lukyanov, Phys. Rev. B **59**, 11163 (1999);  
S. Lukyanov and V. Terras, Nucl. Phys. B **654**, 323 (2003).
- [70] T. Hikihara and A. Furusaki, Phys. Rev. B **69**, 064427 (2004).
- [71] G.S. Uhrig, Phys. Rev. A **80**, 061602(R) (2009).
- [72] A. Mitra and T. Giamarchi, Phys. Rev. Lett. **107**, 150602 (2011);  
A. Mitra and T. Giamarchi, Phys. Rev. B **85**, 075117 (2012).
- [73] A. Mitra, Phys. Rev. Lett. **109**, 260601 (2012);  
A. Mitra, Phys. Rev. B **87**, 205109 (2013).
- [74] E. Perfetto, Phys. Rev. B **74**, 205123 (2006).
- [75] J. Lancaster and A. Mitra, Phys. Rev. E **81**, 061134 (2010);  
T. Sabetta and G. Misguich, Phys. Rev. B **88**, 245114 (2013).
- [76] E. Perfetto and G. Stefanucci, EPL **95**, 10006 (2011).
- [77] J. Rentrop, D. Schuricht, V. Meden, New J. Phys. **14**, 075001 (2012).
- [78] B. Dora, A. Bacsı, and G. Zarand, Phys. Rev. B **86**, 161109 (2012).
- [79] S.A. Hamerla and G.S. Uhrig, New J. Phys. **15**, 073012 (2013).
- [80] F. Pollmann, M. Haque, and B. Dora, Phys. Rev. B **87**, 041109 (2013);  
J.-S. Bernier, R. Citro, C. Kollath, and E. Orignac, Phys. Rev. Lett. **112**, 065301 (2014).
- [81] D. M. Kennes and V. Meden, Phys. Rev. B **88**, 165131 (2013).
- [82] A. Bacsı and B. Dora, Phys. Rev. B **88**, 155115 (2013).
- [83] N. Nessi and A. Iucci, Phys. Rev. B **87**, 085137 (2013).
- [84] S. N. Dinh, D. A. Bagrets, and A. D. Mirlin, Phys. Rev. B **88**, 245405 (2013).
- [85] D.M. Kennes, C. Klöckner and V. Meden, Phys. Rev. Lett. **113**, 116401 (2014).
- [86] R. Sachdeva, T. Nag, A. Agarwal, and A. Dutta, Phys. Rev. B **90**, 045421 (2014).
- [87] S. Sorg, L. Vidmar, L. Pollet, and F. Heidrich-Meisner, Phys. Rev. A **90**, 033606 (2014).
- [88] M. Buchhold and S. Diehl, Phys. Rev. A **92**, 013603 (2015)
- [89] B. Dora and F. Pollmann, arxiv:1410.5954.
- [90] V. E. Korepin, N. M. Bogoliubov and A. G. Izergin, “Quantum Inverse Scattering Method and Correlation Functions”, Cambridge University Press (Cambridge, 1993);  
M. Takahashi, “Thermodynamics of One-Dimensional Solvable Models”, Cambridge University Press (Cambridge, 1999).
- [91] G. Vidal, Phys. Rev. Lett. **98**, 070201 (2007);  
R. Orús and G. Vidal, Phys. Rev. B **78**, 155117 (2008).
- [92] S. R. White and A. E. Feiguin, Phys. Rev. Lett. **93**, 076401 (2004);  
A. J. Daley, C. Kollath, U. Schollwöck, and G. Vidal, J. Stat. Mech. (2004) P04005.
- [93] P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005).
- [94] L. Bonnes, F. H. L. Essler, and A. M. Lauchli, Phys. Rev. Lett. **113**, 187203 (2014).
- [95] J. Mossel and J.-S. Caux, J. Phys. A **45**, 255001 (2012).
- [96] M. Moeckel and S. Kehrein, Phys. Rev. Lett. **100**, 175702 (2008);  
M. Moeckel and S. Kehrein, Ann. Phys. **324**, 2146 (2009).
- [97] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. Adu Smith,

- E. Demler and J. Schmiedmayer, *Science* **337**, 1318 (2012);  
T. Langen, M. Gring, M. Kuhnert, B. Rauer, R. Geiger, D. A. Smith, I. E. Mazets, and J. Schmiedmayer, *Eur. Phys. J. Special Topics* **217**, 43 (2013).
- [98] M. Kollar, F.A. Wolf, and M. Eckstein, *Phys. Rev. B* **84**, 054304 (2011).  
[99] M. Marcuzzi, J. Marino, A. Gambassi, and A. Silva, *Phys. Rev. Lett.* **111**, 197203 (2013).  
[100] F.H.L. Essler, S. Kehrein, S.R. Manmana, and N.J. Robinson, *Phys. Rev. B* **89**, 165104 (2014).  
[101] G.P. Brandino, J.-S. Caux, and R.M. Konik, *arXiv:1407.7167*.  
[102] B. Bertini and M. Fagotti, *J. Stat. Mech.* P07012 (2015).  
[103] B. Bertini, F. H. L. Essler, S. Groha, and N. J. Robinson, *arxiv.org:1506.02994*.